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RESPONSE OF HELICOPTER ROTORS TO PERIODIC FORCES

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ADVANCE RESTRICTED REPORT

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INTRODUCTION

The helicopter depends for sustentation on the downward thrust supplied by a rotor. When in forward flight the rotor is in yaw, with its resultant lift vector inclined to give a horizontal force. This introduces problems not met with in airplanes, unsymmetrical forces on the advancing and retreating airfoils. The Cierva method of dealing with forces not balanced out by periodic change of the blade angle was to introduce flapping hinges, which allow the blade to deflect up and down as the lift force varies. Later these flapping hinges were supplemented by drag hinges, which allow each blade a certain amount of freedom in the plane of rotation so as to take care of the change in angular momentum due to the vertical flapping. Calculations, until recently, have neglected the nonuniformity of downwash over the front and rear portions of the swept disk, and revealed this Coriolis effect as the dominating cause of periodic chordwise bending forces on the blades. But by considering the nonuniformity of downwash over the rotor in forward flight, Seibel (reference 1) has shown that there are large pulsating horizontal forces due to variations of drag, especially evident at low flight speeds, and it is apparently these forces which the drag hinges have served to relieve. This also explains why early attempts to use two-blade rotors resulted in "rough" machines, especially at low flight speeds, while three or four blades seemed to be smooth. This will be explained in detail.

An alternative way of cushioning the shock of horizontal forces is to make the blades rigid, and to introduce flexibility into the pylon which supports the rotor shaft. There are four possible combinations to consider in dealing with horizontal vibrations:

(1) Flexible pylon and flexible or hinged blades

(2) Rigid pylon and flexible or hinged blades

(3) Flexible pylon and rigid blades

(4) Rigid pylon and rigid blades

Combination (1) is poor because it gives a system capable of self-excited oscillations, which can be very destructive. Their theory is dealt with in reference 2.

Combination (2) has often been used, but is complicated by the damping which must be introduced into the blade motions. When the aircraft is on the ground and rocking about on its landing gear, the pylon is no longer effectively rigid, and there is danger of the self-excited phenomenon mentioned under (1). Sufficient damping of the blade hinge motions can remedy this.

Combination (3) is a possible combination which works out well when the natural bending frequencies of pylon and blades are so chosen as to avoid resonance with forced vibrations, as will be explained.

Combination (4) is unsatisfactory. Although the pylon and blades might be sufficiently stiff to avoid resonance, the absence of any flexibility will make the aircraft rough, like a cart without springs being drawn over cobblestones.

The first step in dealing with horizontal vibrations is to find the natural frequencies of vibration of the rotor system. These frequencies are summarized in the section Frequency Equations for Horizontal Vibration, where equations for dealing with various configurations are given, and graphs showing the natural frequencies are plotted. Next, the effects of periodic aerodynamic forces are introduced in the section Horizontal Exciting Forces and summarized in table I. Counterrotating rotors are not dealt with.

In the section Simplified Method of Applying Theory a particular example is worked out to illustrate the method of dealing with any combination of pylon and blade stiffness. It does not represent any existing machine, and is intended merely as a paradigm.

FREQUENCY EQUATIONS FOR HORIZONTAL VIBRATION

The horizontal vibrations of a rotor and pylon are best pictured with the aid of a chart showing the natural frequencies of the system as a function of the rotor revolutions per minute. The fundamental work in determining these natural frequencies for a single rotor was done by Coleman (reference 2), and derivations will not be repeated here. The reader, not content with accepting the frequency equations given here, is referred to reference 2 for complete derivation. The interest here is not in damping, since it has but slight effect on the frequency and is of interest chiefly in the ground resonance phenomenon dealt with in reference 2. A departure from references 2 and 3 also in the choice of coordinates will be apparent below.

NOTATION FOR FREQUENCY EQUATIONS

ω_ϕ angular velocity of blade chordwise vibration when rotor is not turning

a distance from mast center to drag hinge center

For a rigidly attached blade an equivalent value of a can be easily estimated with sufficient accuracy. Cantilever blade in first bending mode is replaced by equivalent system with hinge and spring restraint.

b distance from hinge center to blade center of gravity

r radius of gyration of blade about its center of gravity

ω_a angular velocity of vibration relative to coordinates rotating with the mast

ω angular velocity of mast rotation

ω_p angular velocity of pylon vibration (nonrotating)

n number of blades

m mass of one blade

M equivalent mass (except for blades) concentrated at hub to give observed pylon vibration frequency, assuming strength of spring restraint of pylon is known

ω_f angular velocity of vibration relative to fixed coordinates

ω_x, ω_y angular velocities of vibration of pylon in two fixed directions at right angles to each other. Nonisotropic pylon restraint

(A) Three or more blades. Isotropic pylon restraint.

(1) Rotating coordinates.
Coupled motions:

$$\left\{\left(\omega_\phi^2 + \frac{a}{b\rho^2}\,\omega^2 - \omega_a^2\right)\left[\omega_p^2 - (\omega_a - \omega)^2\right] - \frac{nm(\omega_a - \omega)^4}{2(M+nm)\rho^2}\right\}$$

$$\times\left\{\left(\omega_\phi^2 + \frac{a}{b\rho^2}\,\omega^2 - \omega_a^2\right)\left[\omega_p^2 - (\omega_a + \omega)^2\right] - \frac{nm(\omega_a + \omega)^4}{2(M+nm)\rho^2}\right\} = 0 \quad (1)$$

Uncoupled blade motions:

$$\omega_\phi^2 + \frac{a}{b\rho^2}\,\omega^2 - \omega_a^2 = 0 \quad (2)$$

Uncoupled pylon motions:

$$\omega_p^2 - (\omega_a \pm \omega)^2 = 0 \quad (3)$$

(2) Fixed coordinates.
Coupled motions:

$$\left\{(\omega_p^2 - \omega_f^2)\left[\omega_\phi^2 + \frac{a}{b\rho^2}\,\omega^2 - (\omega_f - \omega)^2\right] - \frac{nm\omega_f^4}{2(M+nm)\rho^2}\right\}$$

$$\times\left\{(\omega_p^2 - \omega_f^2)\left[\omega_\phi^2 + \frac{a}{b\rho^2}\,\omega^2 - (\omega_f + \omega)^2\right] - \frac{nm\omega_f^4}{2(M+nm)\rho^2}\right\} = 0 \quad (4)$$

Uncoupled blade motions:

$$\omega_\phi^2 + \frac{a}{b\rho^2}\,\omega^2 - (\omega_f \pm \omega)^2 = 0 \tag{5}$$

Uncoupled pylon motions:

$$\omega_p^2 = \omega_f^2 \tag{6}$$

(B) Three or more blades. Nonisotropic pylon restraint. Possible in fixed coordinates only.

$$\left\{(\omega_x^2 - \omega_f^2)\left[\omega_\phi^2 + \frac{a}{b\rho^2}\omega^2 - (\omega^2 + \omega_f^2)\right] - \frac{nm\,\omega_f^4}{2(M+nm)\rho^2}\right\}$$

$$\times\left\{(\omega_y^2 - \omega_f^2)\left[\omega_\phi^2 + \frac{a}{b\rho^2}\omega^2 - (\omega^2 + \omega_f^2)\right] - \frac{nm\,\omega_f^4}{2(M+nm)\rho^2}\right\}$$

$$- 4\omega^2\omega_f^2\,(\omega_x^2 - \omega_f^2)\,(\omega_y^2 - \omega_f^2) = 0 \tag{7}$$

Uncoupled motions as in equations (5) and (6).

(C) Two blades. Possible only in rotating coordinates, and with isotropic pylon restraint.

$$\left[\omega_p^2 - (\omega_a + \omega)^2\right]\left[\omega_p^2 - (\omega_a - \omega)^2\right]\left[\omega_\phi^2 + \frac{a}{b\rho^2}\,\omega^2 - \omega_a^2\right]$$

$$- \left[\omega_p^2 - (\omega + \omega_a)^2\right]\frac{nm}{2(M+nm)\rho^2}\,(\omega - \omega_a)^4$$

$$- \left[\omega_p^2 - (\omega - \omega_a)^2\right]\frac{nm}{2(M+nm)\rho^2}\,(\omega + \omega_a)^4 = 0 \tag{8}$$

Uncoupled motions as in equations (2) and (3).

(D) One blade. Isotropic pylon restraint. Rotating coordinates. The unequal inertia characteristics of the blade and its counterweight probably call for a separate analysis by the methods of reference 2 or 3. The two-blade equation can be used, however, as an approximation.

DISCUSSION OF EQUATIONS

Examples of the foregoing equations are plotted in figures 1, 2, 3, and 4. The dash lines show the uncoupled modes. These charts are fundamental in determining the various resonant conditions to expect from a given rotor design. Suppose the rotor of figure 1 is turning at 600 rpm. Then the correct interpretation of the frequency chart is that at any given time the system may not be vibrating at all, or may vibrate <u>at</u> <u>any</u> one of the frequencies represented by A_1, A_2, A_3, or A_4, or any combination of these simultaneously. If the choice of coordinates given for equations (7) and (8) is not followed, there is injected into the answer an added condition that the possible frequencies can appear simultaneously only in pairs or other combinations. This does not result in a frequency chart in the usual sense, and is of no value for purposes of this paper. (An example is fig. 3 of reference 3.) In general, wherever an asymmetry exists, as in the two-blade rotor, or any rotor with nonisotropic pylon restraint, the frequency equation (as commonly interpreted) must be expressed in coordinates in which the directions of asymmetry remain fixed. This calls for rotating coordinates for two-blade rotors, and fixed coordinates for nonisotropic pylon restraint.

HORIZONTAL EXCITING FORCES

Recent work of Seibel (reference 1) gives an explanation of the origin of aerodynamic forces which excite forced vibrations in the horizontal plane. In addition to constant drag force on a blade, there is superposed a pulsating drag force which can be written as

$$\Delta F = a_1 \cos(\psi + \alpha_1) + a_2 \cos 2(\psi + \alpha_2) + a_3 \cos 3(\psi + \alpha_3) + \ldots \tag{9}$$

where

ΔF pulsating force on a blade

ψ blade azimuth angle of position

α_k phase angles

a_k numerical coefficient

It is shown in reference 1 that a_1 has a large value at relatively low airspeeds of the order of about 25 miles per hour, and that at higher airspeeds it rapidly diminishes in magnitude. The first order of excitation is represented by a_1 which means that as the blade makes one complete revolution it receives one complete cycle of impressed force tending to bend it chordwise in its shank. This excitation need not be aerodynamic. It also could be impressed by the gravitational field in a case where a rotor is being tested by rotation about a horizontal axis. The first order also could be excited by the centrifugal force field on a rotor during curvilinear flight. In the usual case the periodic force is aerodynamic, and large only at low airspeeds. Seibel's paper also indicates the possibility of second order, a_2 becoming appreciable in magnitude at higher airspeeds of the order of 70 miles per hour. For the sake of generality, the effect of an hypothetical third order of excitation on various rotors also is included.

It should be emphasized that in a flight condition where any of the coefficients a_k in equation (9) is different from zero each blade will be excited to vibrate in its shank or hinge, and may be in danger of fatigue failure due to large forces if a resonant condition exists. The resultant oscillation of all the blades, however, may or may not make itself felt in the machine as a whole. This depends on whether or not there is a pulsating net force applied to the top of the mast by the vector combination of the forces acting on the individual blades.

If $\Delta F = a_k \cos k(\psi+\alpha_k)$ where k is the order of excitation under consideration, and if equation (9) represents the varying drag force, then, neglecting the phase angle, the horizontal force on the top of the mast is simply the vector sum of the forces due to the individual blades:

$$F_{Hk} = a_k\left\{\cos k\psi e^{i\left(\frac{\pi}{2}+\psi\right)} + \cos k\left(\psi+\frac{2\pi}{n}\right)e^{i\left(\psi+\frac{2\pi}{n}+\frac{\pi}{2}\right)}\right.$$

$$\left. + \ldots + \cos k\left[\psi + \frac{(n-1)2\pi}{n}\right]e^{i\left[\psi+\frac{(n-1)2\pi}{n}+\frac{\pi}{2}\right]}\right\}$$

or

$$F_{Hk} = \frac{a_k e^{i\left(\frac{\pi}{2}+\psi\right)}}{2} \left\{ e^{ik\psi} \left[1 + e^{\frac{i(1+k)2\pi}{n}} + e^{\frac{i2(1+k)2\pi}{n}} + \ldots + e^{\frac{i(n-1)(1+k)2\pi}{n}} \right] + e^{-ik\psi} \left[1 + e^{\frac{i(1-k)2\pi}{n}} + e^{\frac{i2(1-k)2\pi}{n}} + \ldots + e^{\frac{i(n-1)(1-k)2\pi}{n}} \right] \right\}$$

where n is the number of blades.

Term a_1 has a relatively large value at low airspeeds, and a_2 and possibly higher terms may become appreciable at higher speeds. The effects of a_1, a_2, and a_3 on rotors having from one to four blades are shown in table I. They are found by substituting the desired values of k and n in the foregoing equation. In general, there are three types of expression for F_{Hk}, the horizontal force on the top of the mast, expressed above in fixed coordinates.

(1) $F_{Hk} = 0$

(2) F_{Hk} = constant in magnitude and direction

(3) F_{Hk} = a vector the tip of which traces out a curve, frequently a circle, one or more times per mast revolution

Expressions (1) and (2) do not make themselves felt as a vibration in the pilot's cockpit, though they may fatigue the blades individually, while (3) will result in an appreciable disturbance of the whole aircraft unless care is taken to avoid resonance with the natural frequency of the system.

Table I does not show the phase or direction of the force, nor its absolute magnitude, and gives only the effect on various rotors when a_1, a_2, and a_3 are different from zero. Reference in the table to vertical motions is explained under the section Vertical Vibrations.

SIMPLIFIED METHOD OF APPLYING THEORY

Solving the equations of motion with forced vibrations would generally be a very tedious process. To gain a comparative picture of different rotor systems it is much simpler to plot the frequency chart from equation (1) or equation (8) and on the same coordinates draw the straight lines of excitation given at the top of table I. Those straight lines intersect the frequency equations at points of resonance, marking the undesirable values of rotor revolutions per minute.

An example for three blades and isotropic pylon restraint is worked out in figure 3, which shows equations (1), (2), and (3) plotted for

ω_p 200 cycles per minute

ω_ϕ 500 cycles per minute

$$\frac{a}{b\rho^2} \quad 0.2$$

$$\frac{nm}{2(M+nm)\rho^2} = 0.1$$

This last quantity is a measure of the amount of coupling, or the amount by which the frequencies depart from the straight lines and hyperbolas representing pure pylon deflection and pure blade bending, respectively. It is advisable to plot the dash curves from equations (2) and (3) first, and use them as a guide in plotting points from equation (1). Substitute $\omega_a = c\omega$, where c is any convenient constant, in (1), and solutions are easily found. If this is assumed to have been completed, the various danger points illustrated in figure 3 now will be taken up in order.

The frequency chart shows complex roots in the region between A and B. Thus, as in reference 2, values of rotor revolutions per minute between a and b will be dangerous, since self-excited oscillations can occur. This can be remedied by sufficient damping. See reference (2) for details. (If the effective "pylon" is soft, as when an aircraft is rocking on its landing gear, the a - b region may be in the operating range.)

Next, consider the aerodynamic forces at low flight speeds F_{H_1}, represented by the line $\omega_a = \omega$. From the first column, third row of table I, it was found that for three blades the pylon will not be subjected to a vibratory force, and the aircraft will be smooth. From the point of view of individual blade vibration, however, take the intersection of the line $\omega_a = \omega$ with the dash hyperbola plotted from equation (2), find the point C. This gives the value $c = 560$ as a rotor revolutions per minute to be avoided. In spite of the fact that the pilot's cockpit would not feel a vibration, these blades are in a condition of resonance with forced vibration if flown at or near 560 rpm at low airspeeds, and would be in danger of fatigue failure.

The excitation line from the second column of table I is $\omega_a = 2\omega$, and resonance will make the aircraft rough in the three-blade rotors. The points of intersection are shown on figure 3 as D, E, F, and G. The corresponding values of rotor revolutions per minute are at d, e, f, and g.

A vibration corresponding to ordinary shaft whirling occurs at H, the intersection of the line $\omega_a = 0$ with the frequency curves. (In the two-blade case (fig. 2) this point divides into a range of instability similar to A and B of fig. 3.) Thus the rotor of figure 3 would have a satisfactory working range somewhere between $g = 290$ rpm and $c = 560$ rpm and probably would be rough at higher flight speeds if operated below 300 rpm. Figures 1 and 2 show various other combinations which can be considered in rotor design: stiff pylon and flexible blades, and two-blade case. Figure 4 represents the same rotor as figure 3, but referred to nonrotating coordinates. Points A, B, C, and so forth of figure 4 have the same physical significance as the corresponding points of figure 3. The excitation lines cannot be conveniently drawn on figure 4. The line $\omega_a = 2\omega$ transforms into two lines, $\omega_a = 3\omega$ and $\omega_a = \omega$; and $\omega_a = \omega$ transforms into $\omega_a = 2\omega$ and $\omega_a = 0$. An added condition is needed to show which branches of the frequency curves give significant intersections with the various excitation lines, and for this reason it is advised that rotating coordinates (charts similar to fig. 3) be used in practice. Each excitation is represented there by a single line, and it is necessary only to read off the intersections according to the rules given.

The addition of damping terms greatly complicates the equations and generally has only a slight effect on the frequency, though amplitudes of vibration are reduced, and regions of self-excited oscillations are diminished.

The case of a stiff pylon applies only when the aircraft is in flight. On the ground landing-gear flexibility must be considered in calculating ω_p.

NONISOTROPIC PYLON RESTRAINT

Three and Four Blades

A frequency chart referred to rotating coordinates cannot generally be drawn when the pylon frequency is different in different directions. However, with three or more blades the first order of excitation does not excite a coupled mode. Point C, figure 3, is the intersection of the line $\omega_a = \omega$ with the hyperbola from equation (2) and this holds even when $\omega_x \neq \omega_y$; so this resonant point can still be found. With four blades both first and second orders apply to pure blade bending (see table I); so the lines $\omega_a = \omega$ and $\omega_a = 2\omega$ each give only one significant intersection; namely, with the hyperbola of equation (2). In the three-blade case the second-order line $\omega_a = 2\omega$ applies to the coupled modes and tends to make the aircraft rough, and further investigation of this condition with $\omega_x \neq \omega_y$ might be worth while.

For one or two blades the problem with $\omega_x \neq \omega_y$ is laborious to solve in general, though this might be worth while in certain cases. (See reference 5.) Qualitatively, the effect of making the pylon restraint nonisotropic is to reduce the amplitude of forced vibrations near resonance, and at the same time to spread the response over a greater range of rotor revolutions per minute, making the "tuning" less sharp. A chart of ω_a does not have the usual meaning as the motion is not simple harmonic but a combination of different frequencies. A two-blade helicopter seems to be smoother when $\omega_x = \omega_y$ in the flight condition. Vibration isolation is better than when $\omega_x \neq \omega_y$.

VERTICAL VIBRATIONS

A discussion of vertical vibrations by J. A. J. Bennett (reference 4) shows that they make themselves felt in the fuselage only if all blades are given an extra lift impulse simultaneously. For convenience, equation (9) can be considered as applying to periodic vertical forces and table I gives the response to different orders of different numbers of blades.

Consider the lift of a blade element

$$\Delta L \propto \alpha(\omega r + v \sin \psi)^2 \qquad (10)$$

where

ΔL lift of a blade element

α effective (not geometric) angle of attack

r radial position of blade element

v airspeed of machine

The various orders of vertical excitation arise from the fact that α varies periodically around the circle, and the squared bracket varies at first- and second-order frequencies. In addition, the center of lift moves radially along the blade. Thus first, second, and probably higher orders (or values of a_k in equation (9)) are well represented and increase with increasing forward speed. It is beyond the scope of the present paper to discuss the different effects of these excitations on different rotor systems. Much depends on the method of hinging (the amount of δ_3) and on the blade flexibility. The latter can be so chosen that the blade acts as a spring of low natural frequency, and the excitations above first order are only slightly transmitted to the fuselage. In practice a vertical vibration of frequency equal to rotor revolutions per minute usually can be traced to an unsymmetrical condition, such as when one blade is slightly warped, or out of track. An experimental check on the relative magnitudes of higher orders is given in reference 4 for autogiros.

CONCLUSIONS

1. A two-blade rotor not designed to avoid resonance with first-order excitation probably will be noticeably rough at low airspeeds. This condition can be very much improved by proper

choice of pylon and blade stiffness so that the operating revolutions per minute is far from resonance.

2. The first-order roughness will not make itself felt in a three- or four-blade rotor, though each blade may be vibrating individually in its shank, and this condition should be avoided.

3. The second order of excitation also should be avoided; it makes itself felt in the cases of one or three blades but not of two or four.

4. Natural frequency of vertical blade bending should be kept low, and for this reason only a small amount of δ_3 (reduction of blade angle as it flaps upward) can be tolerated, as it has the effect of raising the flapping frequency of a rotating blade. This applies especially to one or two blades, and not so much to three or four.

5. By use of the methods of this report and of reference 1, the chief points of resonance with forced vibrations can be predicted and avoided for any proposed single- or two-rotor system with isotropic pylon restraint.

Bell Aircraft Corporation,
Buffalo, N. Y., Oct. 12, 1944.

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TABLE I

Number of blades	F_{H1} Excitation at low airspeeds $\omega_a = \omega$	F_{H2} Excitation at higher airspeeds $\omega_a = 2\omega$	F_{H3} Excitation $\omega_a = 3\omega$
1	$F_{H1} = a_1 \left(1+e^{2i\psi}\right)$ A circular motion of frequency twice rotor rpm. Resonance will make aircraft rough. Vertical and torquewise disturbance at rotor rpm.	$F_{H2} = a_2 \left(e^{3i\psi}+e^{-i\psi}\right)$ A four-leaf pattern repeated once per rotor revolution. Resonance will make aircraft rough. Also vertical and torquewise disturbance at twice rotor rpm.	$F_{H3} = a_3 \left(e^{4i\psi}+e^{-2i\psi}\right)$ A three-leaf pattern repeated twice per rotor revolution. Resonance will make aircraft rough. Also vertical and torquewise disturbance at three times rotor rpm.
2	$F_{H1} = a_1 \left(1+e^{2i\psi}\right)$ A circular motion of frequency twice rotor rpm. Resonance will make aircraft rough. No net vertical or torquewise excitation.	$F_{H2} = 0$ No horizontal force on mast. Use curve for uncoupled blade vibrations. Vertical and torquewise disturbance at twice rotor rpm.	$F_{H3} = a_3 \left(e^{4i\psi}+e^{-2i\psi}\right)$ A three-leaf pattern repeated twice per rotor revolution. Resonance will make aircraft rough. No net vertical or torquewise excitation.
3	$F_{H1} = a_1$ Constant horizontal force on top of mast. Aircraft not rough. Use curve for uncoupled blade vibrations. No net vertical or torquewise excitation.	$F_{H2} = e^{3i\psi}$ A circular path concentric with mast center of frequency three times rotor rpm. Resonance will make aircraft rough. No net vertical or torquewise excitation.	$F_{H3} = 0$ No horizontal force on top of mast. Use curve for uncoupled blade vibrations. Vertical and torquewise disturbance at three times rotor rpm.
4	$F_{H1} = a_1$ Constant horizontal force on top of mast. Aircraft not rough Use curve for uncoupled blade vibrations. No net vertical or torquewise excitation.	$F_{H2} = 0$ No horizontal force on top of mast. Aircraft not rough. Use curve for uncoupled blade vibrations. No net vertical or torquewise excitation.	$F_{H3} = a_3 e^{4i\psi}$ A circular path concentric with mast center of frequency four times rotor rpm. Resonance will make aircraft rough. No net vertical or torquewise excitation.

Fig. 1. **THREE OR MORE BLADES.**

STIFF PYLON AND FREE HINGES.

$\omega_p = 1000$ cpm

$\omega_\phi = 0$ cpm

Fig. 2. **ONE OR TWO BLADES**

$\omega_p = 200$ cpm

$\omega_\phi = 500$ cpm

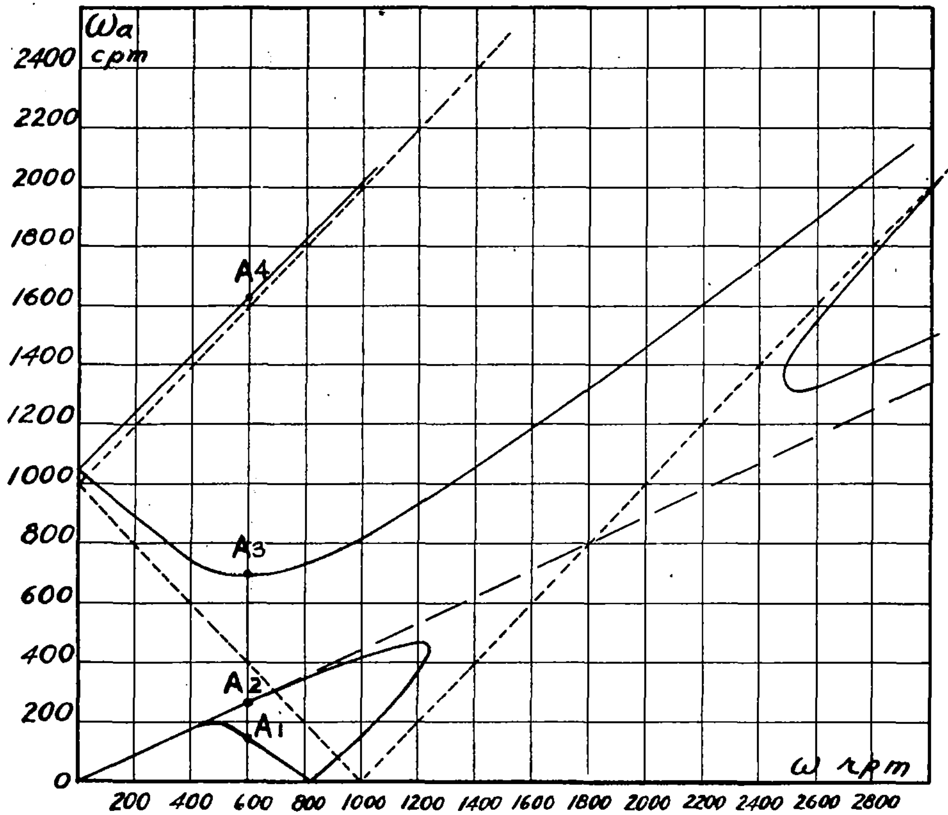

FIGURE 1.

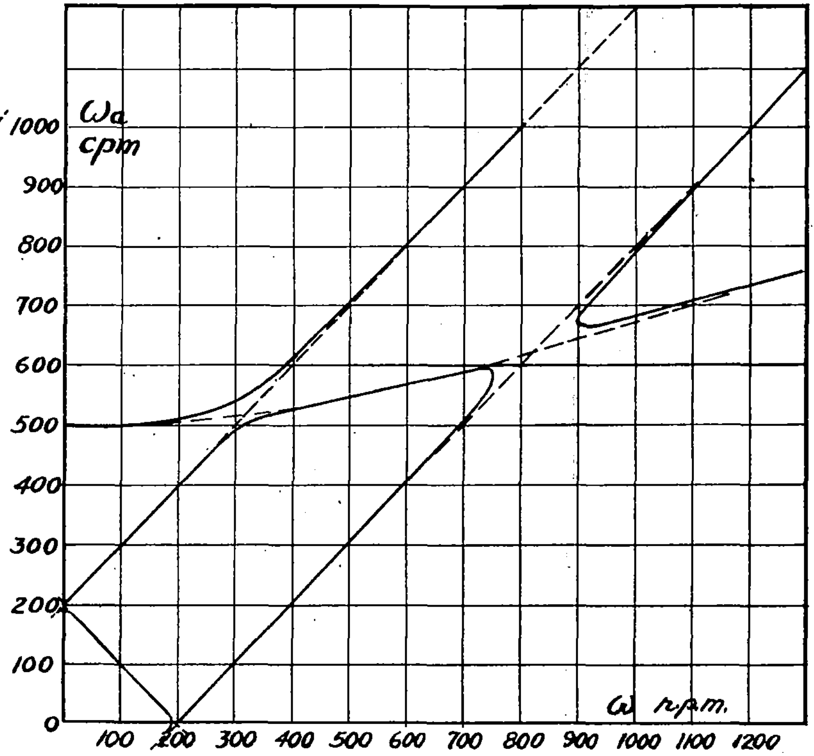

FIGURE 2.

Fig. 3. THREE BLADES

ω_p = 200 cpm

ω_ϕ = 500 cpm

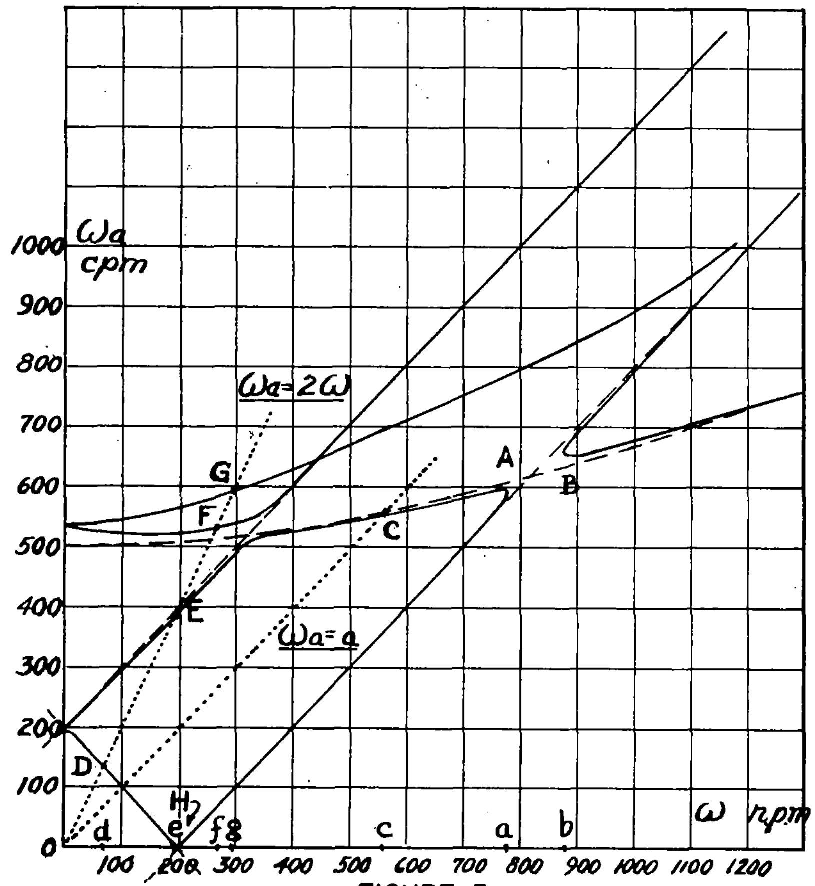

FIGURE 3.

Fig. 4. THREE OR MORE BLADES FIXED CO-ORDINATES.

ω_p = 200 cpm

ω_ϕ = 500 cpm

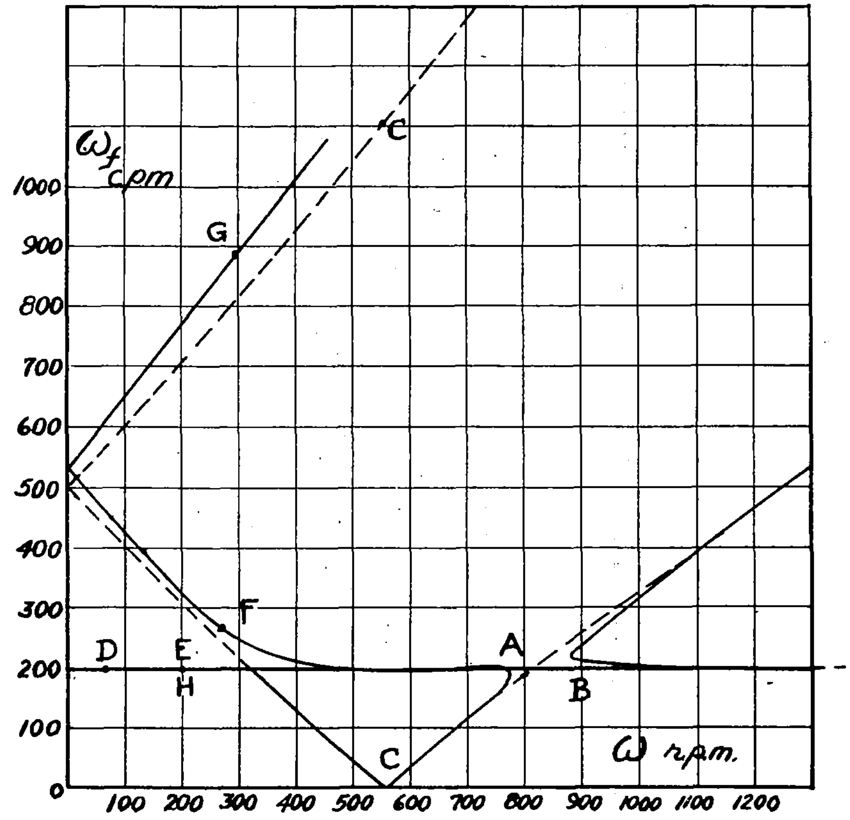

FIGURE 4.